Solutions to Workbook-2 [Mathematics] | Permutation & Combination

Level - 2	**DAILY TUTORIAL SHEET 7**

141.(B) $E=\left[\frac{1}{3}+\frac{1}{50}\right]+\left[\frac{1}{3}+\frac{2}{50}\right]+......\left[\frac{1}{3}+\frac{50}{50}\right]$

If $\frac{x}{50}\geq\frac{2}{3}$ *i.e* $x\geq\frac{100}{3}$ then $\frac{1}{3}+\frac{x}{50}\geq\frac{1}{3}+\frac{2}{3}=1$, If $\frac{x}{50}<\frac{2}{3}$ $\Rightarrow \frac{1}{3}+\frac{x}{50}<1$ $\Rightarrow \left[\frac{1}{3}+\frac{x}{50}\right]=0$

Hence x = 34 for $\left[\frac{1}{3}+\frac{x}{50}\right]\geq 1$ Now, E becomes 1 + 1 + ... 17 times = 17

$E_2(17!)=\left[\frac{17}{2}\right]+\left[\frac{17}{4}\right]+\left[\frac{17}{8}\right]+\left[\frac{17}{16}\right]+\left[\frac{17}{32}\right]$ = 8 + 4 + 2 + 1 + 0 = 15

142.(CD) If one number is 1, then we can choose second from {2 ... 11} in 10 ways
If one number is 2, then we can choose second from {3 ... 12} in 10 ways
If one number is 90, then we can choose second from {91 ... 100} in 10 ways
If one number is 91, then we can choose second from (92 ... 100} in 9 ways
Total ways = (90 × 10) + [9 + 8 + ... + 1] = 945

143.(D) Let A and B be two subsets of S. If $x\in S$, then x will not belong to $A\cap B$ or x belongs to at most one of A, B. This can happen in 3 ways.

Thus, there are $3^4=81$ subsets of S for which $A\cap B=\phi$.

Out of these there is just one way for which $A=B=\phi$

As, we, are interested in unordered pairs of disjoint sets, the number of such unordered pairs is $\frac{1}{2}\left(3^4-1\right)+1=41$

144.(C) Numbers p and q must be of the form $p=r^a\ s^b\ t^c,\ q=r^\alpha\ s^\beta\ t^\gamma$

where $0\leq a,\alpha\leq 2$ and at least one of a,α is 2

$0\leq b,\beta\leq 4$ and at least one of b,β is 4

$0\leq c,\gamma\leq 2$ and at least one of c,γ is 2

Possible values of (a,α) and (c,γ) are (0, 2), (1, 2), (2, 2), (2, 0), (2, 1).

Possible values of (b,β) are (0, 4), (1, 4), (2, 4), (3, 4), (4, 4), (4, 0), (4, 1), (4, 2), (4, 3)

Thus, number of possible ordered pairs (p, q) is $5\times 9\times 5=225$

145.(D) Let $\{x\}=x-[x]$ denote the fractional part of x. Note that $0\leq\{x\}<1$.

We can write the given equation as $\frac{x}{3}-\left\{\frac{x}{3}\right\}+\frac{3x}{2}-\left\{\frac{3x}{2}\right\}+\frac{y}{2}-\left\{\frac{y}{2}\right\}+\frac{3y}{4}-\left\{\frac{3y}{4}\right\}=\frac{11}{6}x+\frac{5}{4}y$

$\Rightarrow \left\{\frac{x}{3}\right\}+\left\{\frac{3x}{2}\right\}+\left\{\frac{y}{2}\right\}+\left\{\frac{3y}{4}\right\}=0$

As each number on the L.H.S. lies in the interval $[0,1)$, we must have

$\left\{\frac{x}{3}\right\}=\left\{\frac{3x}{2}\right\}=\left\{\frac{y}{2}\right\}=\left\{\frac{3y}{4}\right\}=0$ $\Rightarrow$ $\frac{x}{3},\frac{3x}{2},\frac{y}{2}$ and $\frac{3y}{4}$ must be integers.

$\therefore$ x = 6, 12, 18, 24, y = 4, 8, 12, 16, 20, 24, 28

$\Rightarrow$ Number of ordered pairs (x, y) equals $4\times 7=28$

146.(D) Using prime factorization of 1050, we can write the given equation as:

$x_1x_2x_3x_4x_5=2\times 3\times 5^2\times 7$

We can assign 2, 3 or 7 to any of 5 variables. We can assign entire 5^2 to just one variable in 5 ways or can assign $5^2 = 5 \times 5$ to two variables in ${}^5C_2$ ways. Thus, 5^2 can be assigned in ${}^5C_1 + {}^5C_2 = 5 + 10 = 15$ ways

Thus, the required number of solutions is $5 \times 5 \times 5 \times 15 = 1875$

147.(AC) For multiple of 3, either select one from each {1, 4, 7, ..., 298} , {2, 5, 8, ..., 299} , {3, 6, ..., 300} or select all 3 from either. Hence ${}^{150}C_3 \times 3 + 150^3$

For multiple of 2, number of ways is ${}^{150}C_3 + {}^{150}C_1 \times {}^{150}C_2$

148.(B) Total number of ways is equal to total number of ways to select 4 things from 9, that is ${}^9C_4$.

149.(A) Total arrangements = $\frac{9!}{2!3!} = 6 \times 7!$

Undesirable arrangements $= (4!) \times 3!$ {with all balls of same colour occuring together}

Hence, $6(7! - 4!)$

150.(D) Total number of ways $= D_3 \cdot D_3 = 4$

151.(D) Total number of ways is equal to the total number of ways of choosing "m – 1" from "$m - 1 + n - 1$" objects $= {}^{m+n-2}C_{m-1}$

152.(C) **(A)** Total number of selections $= (3+1)(4+1)(2+1) - 1 = 60 - 1 = 59$

(B) Total number of possible sequences $= {}^{10}C_3$

(C) The total number of ways is equal to the total number of integral solutions of $x_1 + x_2 + x_3 + x_4 = 10$ i.e., ${}^{10+4-1}C_{4-1} = {}^{13}C_3 = 286$

(D) The consonants are M, T, H, M, T, C, S and the vowels are A, E, A, I

Group the consonants as one, total number of ways of arrangement $= \frac{5!}{2!} \cdot \frac{7!}{2!2!} = 75600$

153.(B) $x_i - y_i \ge 0$ for all $i = 1,2,3$ and $x_i \ne 0$ as $x_i, y_i \in \{0, 1, 2, 3, ..., 9\}$

For $i = 1,2$

$$\left.\begin{array}{l} \text{Case} - 1 : x_i > y_i - {}^{10}C_2 \text{ ways} \\ \text{Case} - 2 : x_i = y_i - 10 \text{ ways} \end{array}\right\} 55 \text{ ways}$$

For $i = 3 \cdot x_i \ne 0 \ne y_i$

$\left.\begin{array}{l} x_i > y_i - {}^9C_2 \\ x_i = y_i - 9 \end{array}\right\} 45\ ways$, therefore, total number of ways $= (45) \cdot (55)^2$

154.(D) Using P.I.E, the total number of such numbers $= 3^n - {}^3C_1 \cdot 2^n + {}^3C_2 (1)^n$

155.(A) The total number of ways of choosing $A \subset X$ and $B \subset X = 4^{2017}$

The total number of ways to choose $A \cup B = X$ is equal to 3^{2017}.

Total number of ways to select $A \cup B$ to be proper subset of X is equal to say N, where $N = 4^{2017} - 3^{2017}$

Now, total number of elements is N in which (A, B) is the same as (B, A) is equal to $2^{2017} - 1$.

Therefore, total number of required elements

$$= \frac{\left(4^{2017} - 3^{2017}\right) - \left(2^{2017} - 1\right)}{2} + \left(2^{2017} - 1\right) = \frac{\left(4^{2017} - 3^{2017}\right) + \left(2^{2017} - 1\right)}{2}.$$